

Neutrino Mass Models: Impact of non-zero reactor angle

Stephen F. King

*School of Physics and Astronomy, University of Southampton,
Southampton, SO17 1BJ, U.K.*

E-mail: king@soton.ac.uk

ABSTRACT

In this talk neutrino mass models are reviewed and the impact of a non-zero reactor angle and other deviations from tri-bimaximal mixing are discussed. We propose some benchmark models, where the only way to discriminate between them is by high precision neutrino oscillation experiments.

1. Introduction

In the three active neutrino paradigm, the lepton mixing matrix can be parameterised as in Fig.1 in terms of three angles θ_{ij} , one oscillation phase δ and (if neutrinos are Majorana particles) two Majorana phases α_i .

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric

Reactor

Solar

Majorana

Figure 1: The lepton mixing matrix with phases factorizes into a matrix product of four matrices, associated with the physics of Atmospheric neutrino oscillations, Reactor neutrino oscillations, Solar neutrino oscillations and a Majorana phase matrix.

Ignoring the phases, the lepton mixing angles can be visualised as the Euler angles in Fig.2. The mass squared ordering is not yet determined uniquely for the atmospheric mass squared splitting, but the solar neutrino data requires $m_2^2 > m_1^2$, as shown in Fig.3. The absolute scale of neutrino masses is not fixed by oscillation data and the lightest neutrino mass may vary from about 0.0 – 0.2 eV where the upper limit comes from cosmology. The current best fit values for the lepton angles and neutrino mass squared differences are given in Figs.4,5. Note that the reactor angle θ_{13} is not currently measured but its value is only inferred.

2. Why go beyond the Standard Model?

It has been one of the long standing goals of theories of particle physics beyond the Standard Model (SM) to predict quark and lepton masses and mixings. With the discovery of neutrino mass and mixing, this quest has received a massive impetus.

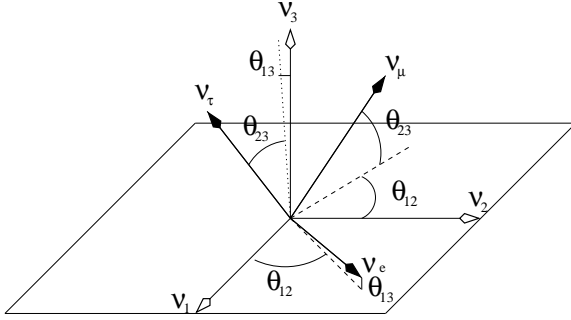


Figure 2: The relation between the neutrino weak eigenstates ν_e , ν_μ , and ν_τ and the neutrino mass eigenstates ν_1 , ν_2 , and ν_3 in terms of the three mixing angles θ_{12} , θ_{13} , θ_{23} . Ignoring phases, these are just the Euler angles representing the rotation of one orthogonal basis into another.

$$\begin{aligned}\theta_{12} &= 34.4 \pm 1.0 \left({}^{+3.2}_{-2.9} \right)^\circ \\ \theta_{23} &= 42.8 {}^{+4.7}_{-2.9} \left({}^{+10.7}_{-7.3} \right)^\circ \\ \theta_{13} &= 5.6 {}^{+3.0}_{-2.7} (\leq 12.5)^\circ\end{aligned}$$

Figure 4: The best fit lepton mixing angles with 1σ error (3σ error).

$$\begin{aligned}\Delta m_{21}^2 &= 7.59 \pm 0.20 \left({}^{+0.61}_{-0.69} \right) \times 10^{-5} \text{ eV}^2 \\ \Delta m_{31}^2 &= \begin{cases} -2.36 \pm 0.11 (\pm 0.37) \times 10^{-3} \text{ eV}^2 \\ +2.46 \pm 0.12 (\pm 0.37) \times 10^{-3} \text{ eV}^2 \end{cases}\end{aligned}$$

Figure 5: The best fit neutrino mass squared differences with 1σ error (and 3σ error).

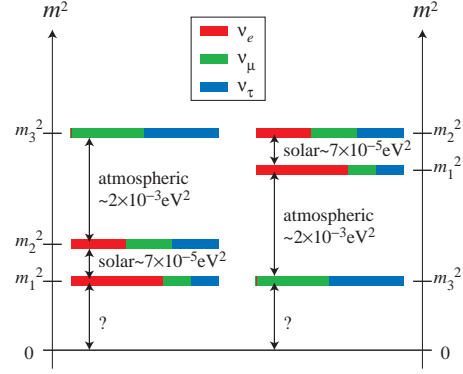


Figure 3: Alternative neutrino mass patterns that are consistent with neutrino oscillation explanations of the atmospheric and solar data. The pattern on the left (right) is called the normal (inverted) pattern. The coloured bands represent the probability of finding a particular weak eigenstate ν_e , ν_μ , and ν_τ in a particular mass eigenstate.

Indeed, perhaps the greatest advance in particle physics over the past decade has been the discovery of neutrino mass and mixing involving two large mixing angles commonly known as the atmospheric angle θ_{23} and the solar angle θ_{12} , while the remaining mixing angle θ_{13} , although unmeasured, is constrained to be relatively small. The largeness of the two large lepton mixing angles contrasts sharply with the smallness of the quark mixing angles, and this observation, together with the smallness of neutrino masses, provides new and tantalizing clues in the search for the origin of quark and lepton flavour. However, before trying to address such questions, it is worth recalling why neutrino mass forces us to go beyond the SM.

Neutrino mass is zero in the SM for three independent reasons:

1. There are no right-handed neutrinos ν_R .
2. There are only Higgs doublets of $SU(2)_L$.

3. There are only renormalizable terms.

In the SM these conditions all apply and so neutrinos are massless with ν_e, ν_μ, ν_τ distinguished by separate lepton numbers L_e, L_μ, L_τ . Neutrinos and antineutrinos are distinguished by total conserved lepton number $L = L_e + L_\mu + L_\tau$. To generate neutrino mass we must relax one or more of these conditions. For example, by adding right-handed neutrinos the Higgs mechanism of the Standard Model can give neutrinos the same type of mass as the electron mass or other charged lepton and quark masses. It is clear that the *status quo* of staying within the SM, as it is usually defined, is not an option, but in what direction should we go?

3. Tri-bimaximal mixing

It is a striking fact that current data on lepton mixing is (approximately) consistent with the so-called tri-bimaximal (TBM) mixing pattern ¹⁾,

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P_{Maj}, \quad (1)$$

where P_{Maj} is the diagonal phase matrix involving the two observable Majorana phases. However in realistic models tri-bimaximal mixing cannot be exact and deviations from tri-bimaximal mixing can be parametrized by three parameters r, s, a defined as ²⁾:

$$\sin \theta_{13} = \frac{r}{\sqrt{2}}, \quad \sin \theta_{12} = \frac{1}{\sqrt{3}}(1 + s), \quad \sin \theta_{23} = \frac{1}{\sqrt{2}}(1 + a). \quad (2)$$

The global fits of the conventional mixing angles ³⁾ can be translated into the 1σ ranges:

$$0.07 < r < 0.21, \quad -0.05 < s < 0.003, \quad -0.09 < a < 0.04. \quad (3)$$

Tri-bimaximal mixing corresponds to $\theta_{12} = 35^\circ$, $\theta_{23} = 45^\circ$ and $\theta_{13} = 0^\circ$. The deviations of the mixing angles from their tri-bimaximal values can be expressed as,

$$\theta_{13} = \Delta_{13}^{TB}, \quad \theta_{12} = 35^\circ + \Delta_{12}^{TB}, \quad \theta_{23} = 45^\circ + \Delta_{23}^{TB}. \quad (4)$$

4. Family Symmetry

Let us expand the neutrino mass matrix in the diagonal charged lepton basis, assuming exact TB mixing, as $M_{TB}^\nu = U_{TB} \text{diag}(m_1, m_2, m_3) U_{TB}^T$ leading to (absorbing the Majorana phases in m_i):

$$M_{TB}^\nu = m_1 \Phi_1 \Phi_1^T + m_2 \Phi_2 \Phi_2^T + m_3 \Phi_3 \Phi_3^T \quad (5)$$

where $\Phi_1^T = \frac{1}{\sqrt{6}}(2, -1, 1)$, $\Phi_2^T = \frac{1}{\sqrt{3}}(1, 1, -1)$, $\Phi_3^T = \frac{1}{\sqrt{2}}(0, 1, 1)$, are the respective columns of U_{TB} and m_i are the physical neutrino masses. In the neutrino flavour basis (i.e. diagonal charged lepton mass basis), it has been shown that the above TB neutrino mass matrix is invariant under S, U transformations:

$$M_{TB}^\nu = SM_{TB}^\nu S^T = UM_{TB}^\nu U^T. \quad (6)$$

A very straightforward argument ⁴⁾ shows that this neutrino flavour symmetry group has only four elements corresponding to Klein's four-group $Z_2^S \times Z_2^U$. By contrast the diagonal charged lepton mass matrix (in this basis) satisfies a diagonal phase symmetry T . The matrices S, T, U form the generators of the group S_4 in the triplet representation, while the A_4 subgroup is generated by S, T . Some candidate family symmetries G_f are shown in Fig.6.

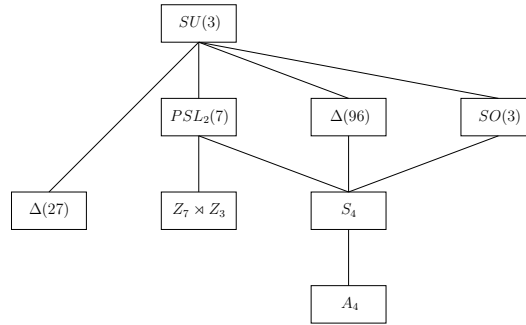


Figure 6: Some subgroups of $SU(3)$ that contain triplet representations and have been used as candidate family symmetries G_f .

5. Direct vs Indirect Models and Form Dominance

As discussed in ⁴⁾, the flavour symmetry of the neutrino mass matrix may originate from two quite distinct classes of models. The first class of models, which we call direct models, are based on a family symmetry $G_f = S_4$, or a closely related family symmetry as discussed below, some of whose generators are directly preserved in the lepton sector and are manifested as part of the observed flavour symmetry. The second class of models, which we call indirect models, are based on some more general family symmetry G_f which is completely broken in the neutrino sector, while the observed neutrino flavour symmetry $Z_2^S \times Z_2^U$ in the neutrino flavour basis emerges as an accidental symmetry which is an indirect effect of the family symmetry G_f . In such indirect models the flavons responsible for the neutrino masses break G_f completely so that none of the generators of G_f survive in the observed flavour symmetry $Z_2^S \times Z_2^U$.

In the direct models, the symmetry of the neutrino mass matrix in the neutrino flavour basis (henceforth called the neutrino mass matrix for brevity) is a remnant of the $G_f = S_4$ symmetry of the Lagrangian, where the generators S, U are preserved in the neutrino sector, while the diagonal generator T is preserved in the charged

lepton sector. For direct models, a larger family symmetry G_f which contains S_4 as a subgroup is also possible e.g. $G_f = PSL(2, 7)$ ⁵⁾. Typically direct models satisfy form dominance ^{6,7)}, and require flavon F-term vacuum alignment, permitting an $SU(5)$ type unification ⁸⁾ typically based in A_4 family symmetry ⁹⁾. Such minimal A_4 models lead to neutrino mass sum rules between the three masses m_i , resulting in/from a simplified mass matrix in Eq.5. A_4 may result from 6D orbifold models ¹⁰⁾ and recently an $A_4 \times SU(5)$ SUSY GUT model has been constructed in 6D ¹¹⁾, while a similar model in 8D enables vacuum alignment to be elegantly achieved by boundary conditions ¹²⁾.

In the indirect models ⁴⁾ the idea is that the three columns of U_{TB} Φ_i are promoted to new Higgs fields called “flavons” whose VEVs break the family symmetry, with the particular vacuum alignments along the directions Φ_i . In the indirect models the underlying family symmetry of the Lagrangian G_f is completely broken, and the flavour symmetry of the neutrino mass matrix $Z_2^S \times Z_2^U$ emerges entirely as an accidental symmetry, due to the presence of flavons with particular vacuum alignments proportional to the columns of U_{TB} , where such flavons only appear quadratically in effective Majorana Lagrangian ⁴⁾. Such vacuum alignments can be elegantly achieved using D-term vacuum alignment, which allows the large classes of discrete family symmetry G_f , namely the $\Delta(3n^2)$ and $\Delta(6n^2)$ groups ⁴⁾. The indirect models satisfy natural form dominance since each column of the Dirac mass matrix corresponds to a different flavon VEV. In the limit $m_1 \ll m_2 < m_3$ FD reduces to constrained sequential dominance (CSD) ¹³⁾. Examples of discrete symmetries used in the indirect approach can be found in ¹⁴⁾.

Explicitly, the TB form of the neutrino mass matrix in Eq.5 is obtained from the see-saw mechanism in these models as follows. In the diagonal right-handed neutrino mass basis we may write $M_{RR}^\nu = \text{diag}(M_A, M_B, M_C)$ and the Dirac mass matrix as $M_{LR}^\nu = (A, B, C)$ where A, B, C are three column vectors. Then the type I see-saw formula $M^\nu = M_{LR}^\nu (M_{RR}^\nu)^{-1} (M_{LR}^\nu)^T$ gives

$$M^\nu = \frac{AA^T}{M_A} + \frac{BB^T}{M_B} + \frac{CC^T}{M_C}. \quad (7)$$

By comparing Eq.7 to the TB form in Eq.5 it is clear that TB mixing will be achieved if $A \propto \Phi_3$, $B \propto \Phi_2$, $C \propto \Phi_1$, with each of $m_{3,2,1}$ originating from a particular right-handed neutrino of mass $M_{A,B,C}$, respectively. This mechanism allows a completely general neutrino mass spectrum and, since the resulting M^ν is form diagonalizable, it is referred to as form dominance (FD) ⁶⁾. For example, the direct A_4 see-saw models ⁸⁾ satisfy FD ⁶⁾, where each column corresponds to a linear combination of flavon VEVs.

A more natural possibility, called Natural FD, arises when each column arises from a separate flavon VEV, and this possibility corresponds to the case of indirect models. For example, if $m_1 \ll m_2 < m_3$ then the precise form of C becomes irrelevant, and

in this case FD reduces to constrained sequential dominance (CSD)¹³⁾. The CSD mechanism has been applied in this case to the class of indirect models with Natural FD based on the family symmetries $SO(3)$ ^{13,16)} and $SU(3)$ ¹⁵⁾, and their discrete subgroups¹⁴⁾.

6. Tri-bimaximal mixing and GUTs

Tri-bimaximal mixing (even if only approximately realised) seems to suggest an underlying non-Abelian discrete family symmetry such as S_4 that might unlock the long-standing flavour puzzle. However, when combined with Grand Unification such as $SU(5)$, the tri-bimaximal mixing prediction is always violated due to the requirement of non-zero quark mixing^{17,18,19,20,21)}. The resulting mixing from the charged lepton sector will always lead to deviations from tri-bimaximal lepton mixing, for example resulting in a non-zero reactor angle of about $\theta_{13}^o \approx 3^\circ$. *Note that the reactor angle cannot be equal to zero in $TBM \otimes GUT$ models.*

Moreover, $TBM \otimes GUT$ models predict sum rule relations between the deviation parameters such as^{13,22,23,24,25)}.

$$\Delta_{12}^{TB} \approx \theta_{13} \cos \delta, \quad \theta_{13} \approx \frac{\theta_C}{3\sqrt{2}} \approx 3^\circ \quad (8)$$

where $\theta_C = 13^\circ$ is the Cabibbo angle and δ is the observable CP violating oscillation phase, with RG corrections of less than one degree. Such sum rules provide a motivation for future high precision neutrino oscillation experiments capable of measuring the reactor angle and the CP violating oscillation phase, as well as the deviation of the solar angle from its tri-bimaximal value²⁶⁾.

In certain classes of $TBM \otimes GUT$ models, the leptonic CP violating phase δ is related to the CKM unitarity triangle angle $\alpha \approx 90^\circ$. In practice the Family Symmetry $\otimes GUT$ models which predict $\alpha = 90^\circ$ also predict a discrete set of possibilities for the CP violating oscillation phase $\delta = 0^\circ, 90^\circ, 180^\circ, 270^\circ$ ²⁷⁾. These discrete possibilities could be distinguished by future high precision neutrino oscillation experiments.

7. Tri-bimaximal-reactor Mixing

If the reactor angle is measured to be large, but the solar and atmospheric angles remain close to their tri-bimaximal values, i.e. the deviation parameters in Eq.2 take the form $s = a = 0$ but $r \neq 0$, then the mixing matrix takes the “tri-bimaximal-reactor” (TBR) form²⁸⁾:

$$U_{TBR} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}}(1-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}} \end{pmatrix} P. \quad (9)$$

Such TBR pattern may be accomplished via partially constrained sequential dominance (PCSD) as a variation of Eq.7 in the hierarchical limit where $m_1 \ll m_2$ where C is irrelevant and by taking $B^T = \frac{1}{\sqrt{3}}(1, 1, -1)$, $A^T = \frac{1}{\sqrt{2}}(\varepsilon, 1, 1)$, where ε is a small correction to the vacuum alignment, leading to $\varepsilon = re^{-i\delta}$ ²⁸⁾.

8. Quark-Lepton Complementarity

If the reactor angle is measured to be larger than the above prediction and the solar and atmospheric angles deviate significantly from their TB values, then one may consider as a starting point bimaximal mixing which corresponds to $\theta_{12} = 45^\circ$, $\theta_{23} = 45^\circ$ and $\theta_{13} = 0^\circ$. Non-Abelian discrete family symmetry models based on S_4 can alternatively lead to bimaximal mixing if the requirement of GUTs is relaxed ²⁹⁾. Such models require large deviations and these large deviations imply a large reactor angle, typically $\theta_{13} \approx \theta_C \approx 13^\circ$.

The deviations of the mixing angles from their bimaximal values can be expressed as,

$$\theta_{12} = 45^\circ + \Delta_{12}^{BM}, \quad \theta_{23} = 45^\circ + \Delta_{23}^{BM}. \quad (10)$$

The experimentally measured solar angle requires a large deviation corresponding to the sum rule relation ²³⁾,

$$\Delta_{12}^{BM} \approx \theta_{13} \cos \delta, \quad \theta_{13} \approx \theta_C. \quad (11)$$

Assuming $\delta \approx 180^\circ$, this would imply,

$$\theta_{12} + \theta_C = 45^\circ, \quad (12)$$

known as “quark-lepton complementarity” (QLC). There is no straightforward GUT model that can achieve QLC.

9. Abelian Family Symmetry

With a simple $U(1)$ family symmetry it is impossible to obtain exact tri-bimaximal or bimaximal mixing, except by accident. On the other hand it is easy to explain qualitative features such as,

$$\theta_{12} \sim \text{large}, \quad \theta_{23} \sim \text{large}, \quad \theta_{13} \sim \text{small}. \quad (13)$$

For example, for a hierarchical spectrum the typical expectation of see-saw models with sequential dominance models approximately given by ³⁰⁾

$$\theta_{13} \sim O\left(\frac{m_2}{m_3}\right). \quad (14)$$

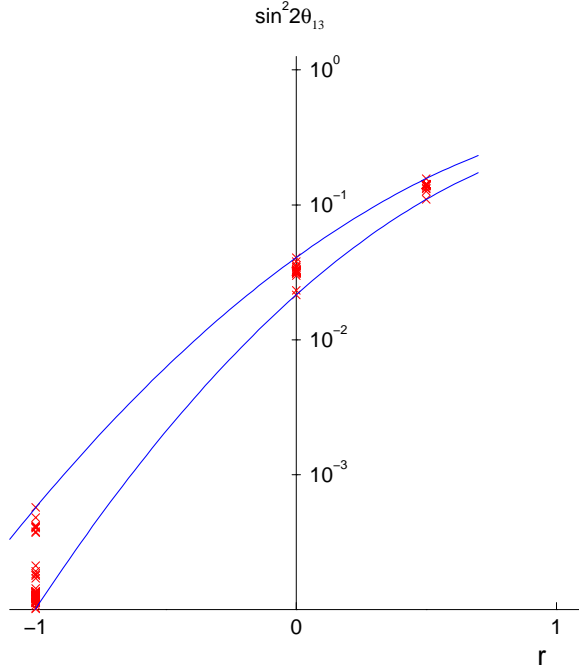


Figure 7: The prediction for $\sin^2 2\theta_{13}$ in Abelian family symmetry models (normal hierarchy) as a function of an undetermined ratio of Yukawa couplings r .

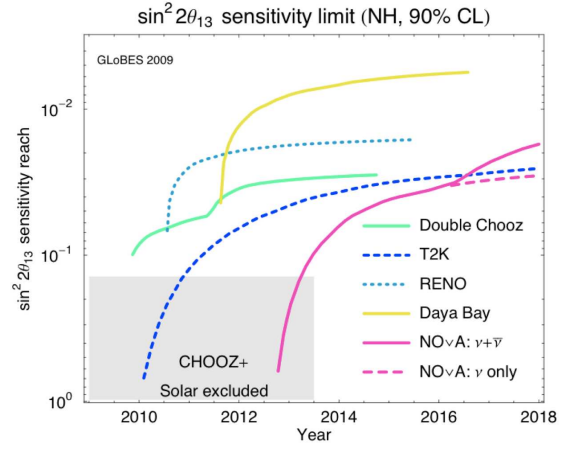


Figure 8: The future $\sin^2 2\theta_{13}$ sensitivity limit (normal hierarchy, 90% CL).

However the precise prediction depends on an undetermined ratio of Yukawa couplings r as shown in Fig.7, which may be compared to the future experimental sensitivities in Fig.8³¹⁾.

10. Summary

Over the past dozen years there has been a revolution in our understanding of neutrino physics. Yet, despite this progress, it must be admitted that we still do not understand the origin or nature of neutrino mass and mixing. However it is a striking fact that current data on lepton mixing is consistent with the so-called tri-bimaximal mixing pattern and many models have been proposed. Realistic models predict various deviations from tri-bimaximal mixing, for example the benchmark models shown in Table 1.

In order to discriminate between the benchmark models in Table 1, and hence shed light on GUT models of Flavour, it is necessary to measure the deviations of the reactor, solar and atmospheric angles from their tri-bimaximal values, as well as δ . From a theorists' perspective the job is not done until the deviations from tri-bimaximal mixing and δ are measured. This will require high precision neutrino oscillation experiments, based on a next generation neutrino accelerator²⁶⁾.

Benchmark Model	θ_{13}	$ \theta_{23} - 45^\circ $	$ \theta_{12} - 35^\circ $	δ
TBM \otimes GUT ²⁷⁾	$\frac{\theta_C}{3\sqrt{2}} = 3^\circ$	$\leq 1^\circ$	$\leq 1^\circ$	$90^\circ, 270^\circ$
TBR ²⁸⁾	any	$\leq 1^\circ$	$\leq 1^\circ$	any
QLC ²⁹⁾	$\theta_C = 13^\circ$	$\leq 1^\circ$	large	180°
Abelian ³⁰⁾	Fig.7	large	large	any

Table 1: Predictions of benchmark models for the deviations from tri-bimaximal mixing and δ .

Postscript: while writing these proceedings T2K have published evidence for a large non-zero reactor angle ³²⁾. If confirmed these results would have major implications for neutrino mass models as discussed above.

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